Student Number:
Teacher:

St George Girls High School

Mathematics Advanced

2024 Trial HSC Examination

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in **Section I**, use the Multiple-Choice answer sheet provided at the back of this booklet.
  For questions in **Section II:**
  - Answer the question in the spaces provided.
  - **Show relevant mathematical reasoning** and/or calculations.
  - **Extra writing space** is provided at the back of each booklet. If you use this space, clearly indicate which question you are answering.
  - Marks may not be awarded for incomplete or poorly presented solutions or where multiple solutions are provided.

Total marks: 100

Section I – 10 marks (pages 2 to 7)

- Attempt Questions 1-10
- Allow about 20 minutes for this section.

Section II – 90 marks (pages 9 to 40)

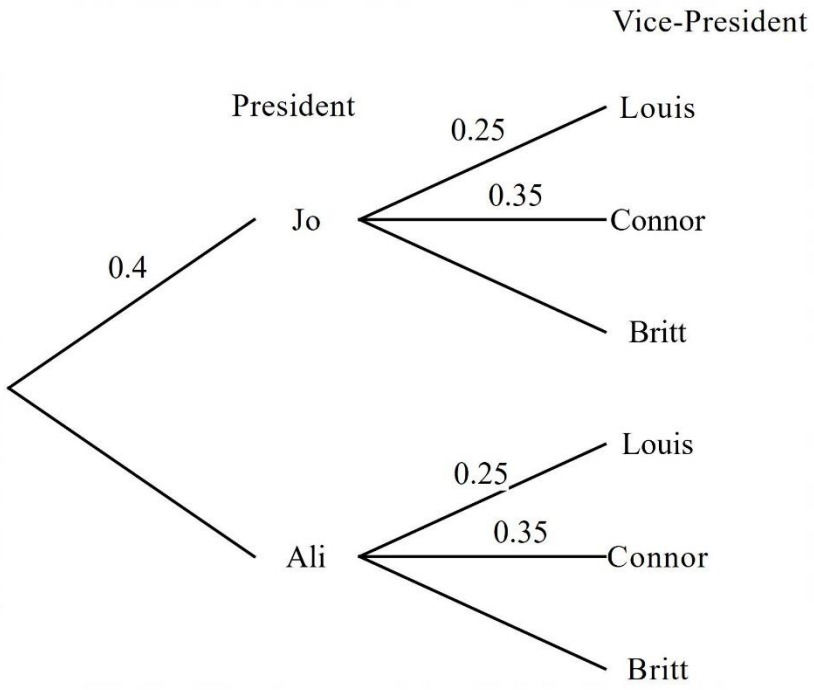
- Attempt Questions 11 – 35
- Allow about 2 hours and 40 minutes for this section.

Q1 – Q10	/10
Q11 – Q17	/20
Q18 – Q23	/18
Q24– Q28	/17
Q29 – Q32	/17
Q33 – Q35	/18
Total	**/100**
	%

Section I

10 marks
Attempt Questions 1 – 10.
Allow about **20 minutes** for this section.
Use the **multiple-choice answer sheet** for questions 1 – 10.

1. The partially completed probability tree below shows the results of a survey of voting intentions of shareholders in an election for President and Vice-President of a large corporation.



Based on this survey, what is the probability that Jo is elected President, with either Britt or Louis as Vice-President?

(A) 0.04

(B) 0.26

(C) 0.30

(D) 0.50

2. Which one of the following statements about the line $12x - 4y = 0$ is **NOT** true?

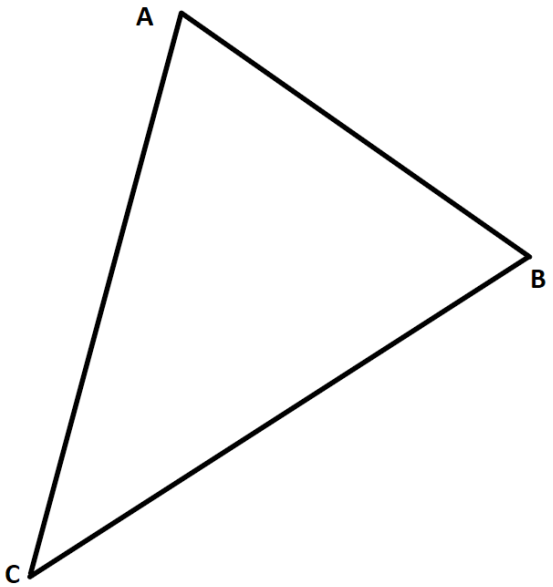
(A) The line has a slope of 12

(B) The line passes through the origin

(C) The line has the same slope as the line with the equation $12x - 4y = 12$

(D) For this line, as x increases y increases

3. On her bushwalk, Angela walks 2.5 km from A to B on a bearing of 140° and then walks from B to C for 3.5 km on a bearing of 235°.



How far, to one decimal place, does Angela need to walk to return to the starting point?

(A) 5.2 km

(B) 4.4 km

(C) 4.1 km

(D) 3.1 km

4. The parabola $f(x) = -x^2 + 9mx + 2$ has a maximum at $x = \frac{2}{m}$, where m is a constant such that $m \neq 0$.

What are the values of m?

(A) $m = \frac{2}{3}$ or $m = -\frac{2}{3}$

(B) $m = \frac{2}{9}$ or $m = -\frac{2}{9}$

(C) $m = \frac{9}{4}$ or $m = -\frac{9}{4}$

(D) $m = \frac{3}{2}$ or $m = -\frac{3}{2}$

5. What are the coordinates of the turning point to the curve $y = e^x - ex$?

(A) $(0, 1)$

(B) $(e, 1)$

(C) $(1, e)$

(D) $(1, 0)$

6. A series has following terms:

2+5+8+11+14+17+ …

What is the sum of the first 30 terms of the series

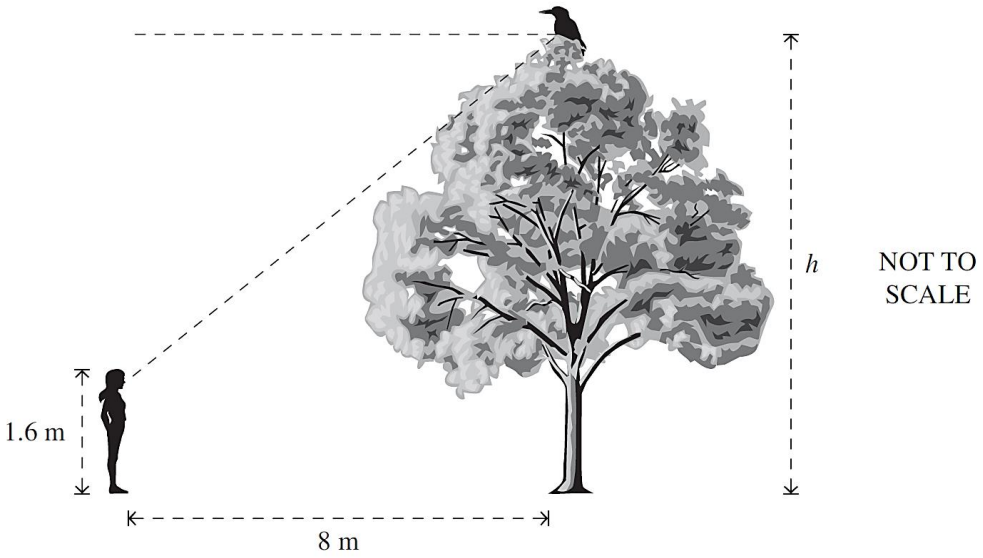
(A) 1410

(B) 1365

(C) 1380

(D) 2820

7. Kelly is watching a bird at the top of a tree. Kelly is 1.6 m tall and is standing 8 m away from the tree, as shown in the diagram.



If the angle of depression of Kelly from the bird is 73°, what is the height of the tree?

(A) 2.45 m

(B) 4.05 m

(C) 26.17 m

(D) 27.77 m

8. The functions $g(x)$ and $h(x)$ are defined as follows:

$$g(x) = \frac{x^2}{8} \qquad h(x) = \sqrt{2x} + 3$$

Which is the correct expression for $h\big(g(x)\big)$?

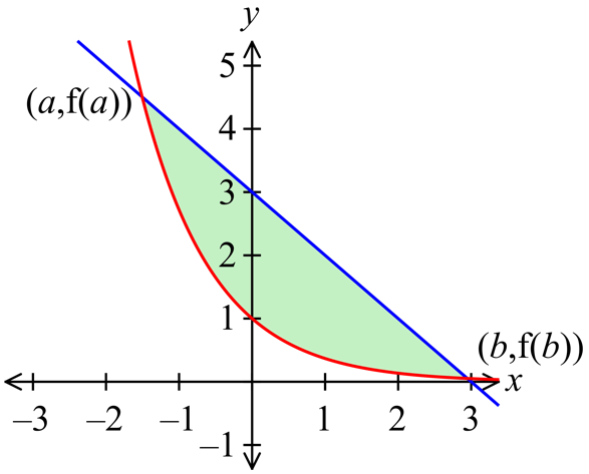
(A) $\sqrt{\frac{2x}{3}}$

(B) $\frac{x}{2} - 3$

(C) $\frac{x}{2} + 3$

(D) $\sqrt{\frac{x}{2} + 3}$

9. The diagram below shows the region enclosed by $x + y = 3$ and $y = e^{-x}$.



Which of the following integrals represents the area of the shaded region?

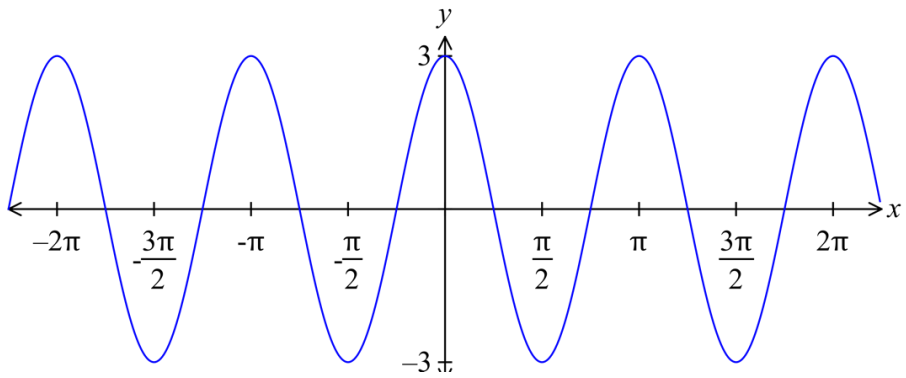
(A) $\int_a^b (e^{-x} - 3 + x)dx$

(B) $\int_a^b (3 - x + e^{-x})dx$

(C) $\int_a^b (e^{-x} + 3 + x)dx$

(D) $\int_a^b (3 - x - e^{-x})dx$

10. The graph of $y = f(x)$ is show below.



Which of the following could **NOT** be the correct description of $f(x)$?

(A) $y = 3\sin 2(x + \frac{\pi}{2})$

(B) $y = 3\sin(2x + \frac{\pi}{2})$

(C) $y = 3\cos 2x$

(D) $y = 3\cos 2(x - \pi)$

END OF SECTION I

Mathematics Advanced

Section II Answer Booklet 1

Student Number:
Teacher:

Section II
90 marks
Attempt Questions 11 – 35
Allow about 2 hours 40 minutes for this section

Booklet 1 – Attempt Question 11 – 28 (55 marks)
Booklet 2 – Attempt Question 29 – 35 (35 marks)

Instructions

- Write your Teacher's Name and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 26 & 27 of Booklet 1. If you use this space, clearly indicate which question you are answering.

Q11 – Q17	/20
Q18 – Q23	/18
Q24 – Q28	/17

Please turn over

Section II

90 marks
Attempt questions 11 - 35
Allow about 2 hours and 40 minutes for this section
Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks) **Marks**

Solve $\log_3 5 = 2\log_3 10 - \log_3 x$. **2**

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Question 12 (2 marks) **Marks**

Find the coordinates of the centre and the radius of the circle with equation **2**

$$x^2 + 8x + y^2 - 6y - 24 = 0$$

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Question 13 (2 marks)

A Year 12 class of 24 students were surveyed about the type of exercise they do.

12 go to the gym, 13 run and 4 neither go to the gym nor run.

(a) Complete the two-way table below to represent this data. **1**

	Gym	**Do Not Gym**	**Total**
Run			13
Do Not Run		4	
Total	12		

(b) Find the probability that a student who runs also goes to the gym. **1**

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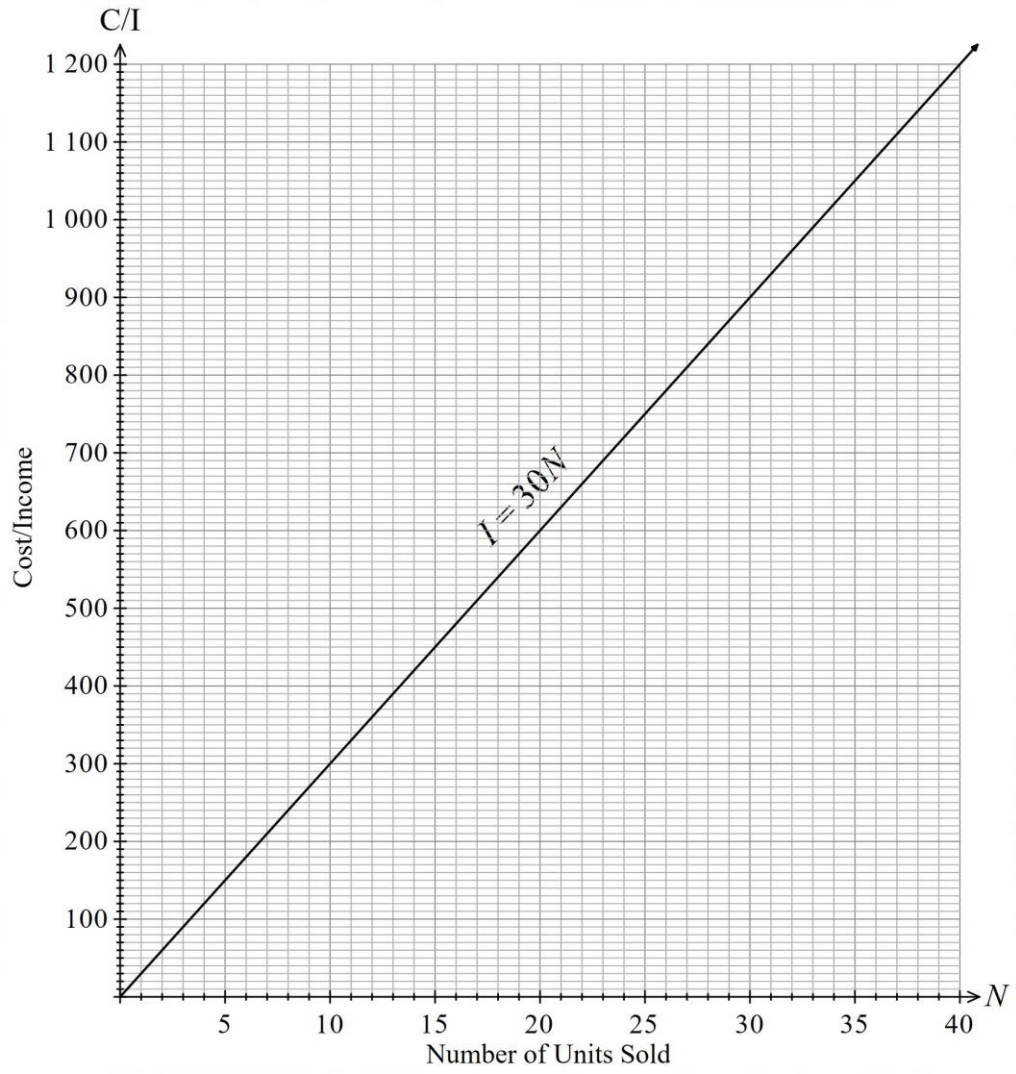
Question 14 (5 marks) **Marks**

A wholesaler sells a device called a Midien for \$30 each. The income I from selling N devices is graphed below.



Each day their fixed costs (wages, rent etc) are \$300 and each Midien costs them \$10.

(a) The formula for the cost involved in selling N Midiens in a day is $C = 10N + 300$. What is the cost when 20 Midiens are sold in a day? **1**

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Question 14 continues on page 13

Question 14 (continued) **Marks**

(b) Draw the line representing the equation $C = 10N + 300$ on the graph on page 12. **2**

(c) How many Midiens would they need to sell in a day to break even? **1**

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(d) How much profit or loss would they make on a day where they sold 30 Midiens? Clearly state profit or loss. **1**

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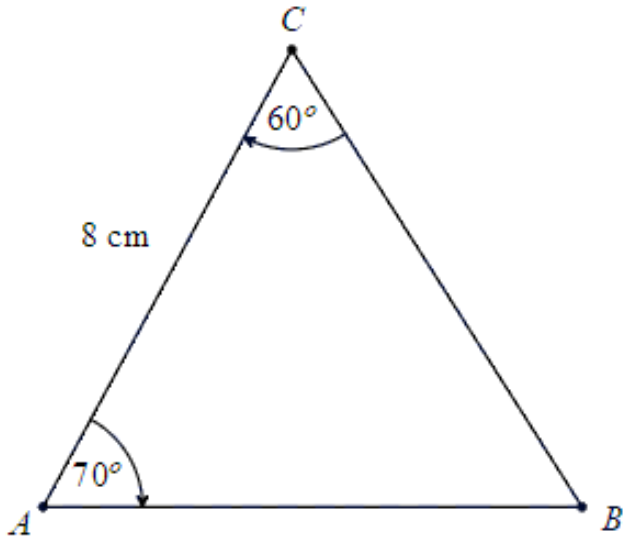
Question 15 (3 marks) **Marks**

In the diagram below it is given that $AC = 8\,cm$, $\angle C = 60^\circ$ and $\angle A = 70^\circ$.



(a) Show that the length of AB is 9.04 cm. **2**

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(b) Hence, find the area of the triangle to 2 decimal places. **1**

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Question 16 (3 marks)

The number of people (N) who attend a show at a venue varies inversely with the amount of floor space in cm^2 allowed per person (A). Three hundred people attended the venue when $1m^2$ of floor space was allowed per person.

(a) Find the constant (k) of variation. **1**

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(b) Hence or otherwise, find how many people can attend the venue if each person is allowed 4500 cm^2 of floor space. **2**

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Question 17 (3 marks)

Consider the series $\ln x - 3(\ln x)^2 + 9(\ln x)^3 + \cdots$

(a) Show that these terms form a geometric series. **1**

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(b) Hence, or otherwise, find the values of x for which the series has a limiting sum. **2**

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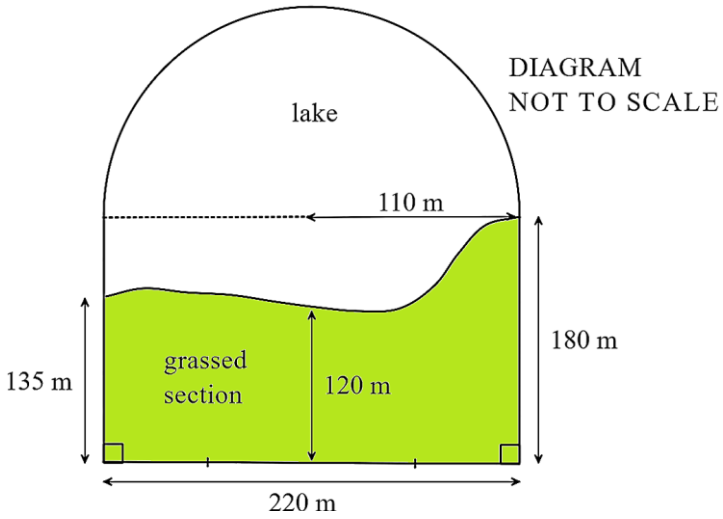
Question 18 (3 marks)

A landscaper, Annabel, wanted to ask the local council to renovate the park near her home. Her suggestion was to have a park partially occupied by a lake and the rest be a grassed section, as shown in the diagram below.



The park consists of a rectangle with dimensions 220 m and 180 m, and a semi-circle with a radius of 110 m. Some measurements from the end of the grassed section to the edge of the lake are also shown.

Using two applications of the Trapezoidal rule, calculate the approximate area of the lake, to the nearest square metre. **3**

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Question 19 (3 marks)

The third term in a geometric sequence 18 and the sixth term is 486.

(a) Find the first term and the common ratio. **2**

(b) Find the sum of first 10 terms of this sequence. **1**

Question 20 (3 marks)

Find the equation of the normal to the curve $y = 3\log_e 2x$, when it passes through the point P whose $x-$ coordinate is 2. Leave your answer in the form:

$$y - y_1 = m(x - x_1)$$

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Question 21 (2 marks) **Marks**

Given that $\int_1^6 f(x)dx = 6$ and $\int_3^6 f(x)dx = -4$. What is the value of $\int_1^3 (f(x) + x)dx$? **2**

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Question 22 (2 marks)

Differentiate $y = \left(1 + e^{2x}\right)^8$. **2**

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Question 23 (5 marks) **Marks**

An object moves along the number line so that its displacement (x metres) from the origin is given by:

$$x(t) = 12 - 2t + 12\log_e(t+3)$$

where time t is measured in seconds.

(a) Where is the particle initially? **1**

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(b) Show that the particle is at rest when $t = 3$. **2**

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(c) In which direction is the particle moving when $t > 3$? Justify your statement with mathematical working or in words. **2**

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Question 24 (3 marks) **Marks**

(a) Show that $\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A} = \sec^2 A$. **2**

(b) Hence, or otherwise, find $\int \tan^2 x \left(\dfrac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x}\right) dx$. **1**

Question 25 (3 marks) **Marks**

Evaluate $\int_1^3 \frac{3x^2-1}{5x^3-5x+9}\,dx$ leaving your answer in simplest exact form. **3**

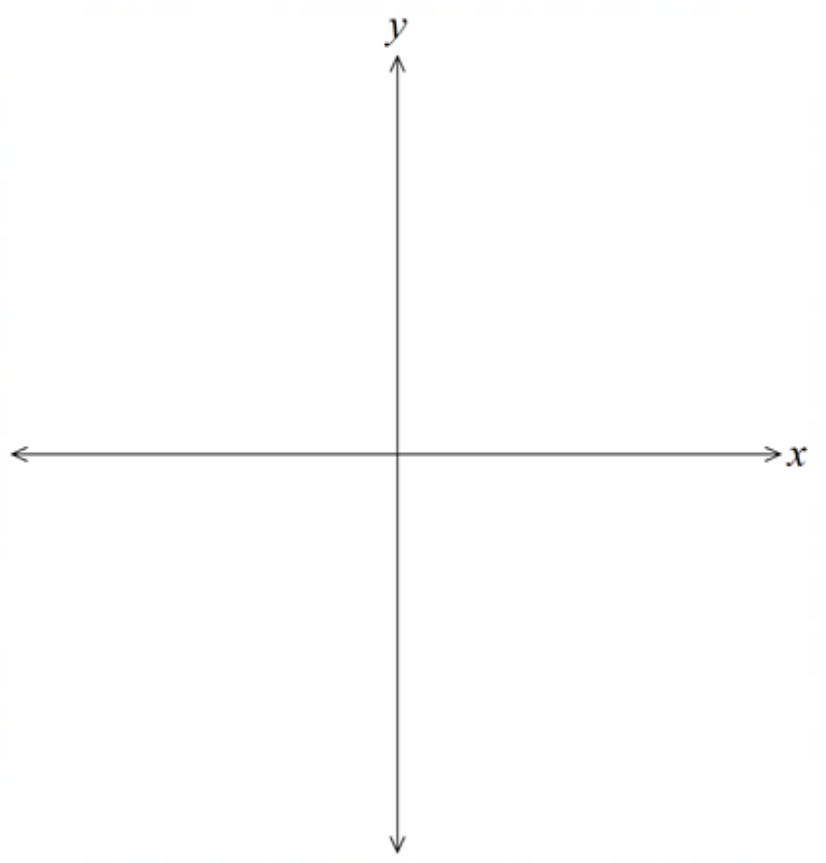
Question 26 (2 marks)

If $\sec^2\theta = 2$ and $0 \leq \theta \leq \pi$, find all possible values of θ. **2**

Question 27 (5 marks) **Marks**

(a) Let $f(x) = (x+2)^2 - 1$

Plot the graph of $f(x)$ in the given space below, showing all intercepts and turning point. **2**



(b) $f(x)$ is transformed to $g(x)$ such that,

$$g(x) = 2f(3-x)$$

Describe the three transformations in the correct order. **3**

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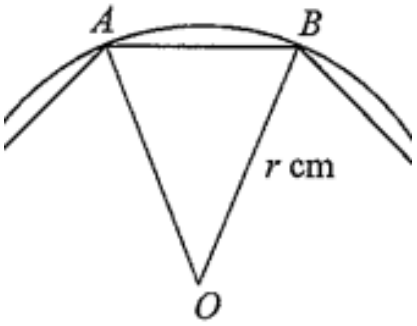
Question 28 (4 marks) **Marks**

In the diagram ΔAOB is a section of a regular ten sided decagon inscribed in a circle with centre O and radius r cm.



(a) Find r given that the area of the full decagon is 100 cm^2. **2**

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(b) Find the perimeter of the decagon. Give your answer correct to 3 decimal places. **2**

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END OF SECTION II

Section I - Extra writing space

If you use this space, clearly indicate which question you are answering.

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Section I - Extra writing space

If you use this space, clearly indicate which question you are answering.

Mathematics Advanced

Section II Answer Booklet 2

Student Number:
Teacher:

Section II

Booklet 2 – Attempt Question 29 – 35 (35 marks)

Instructions

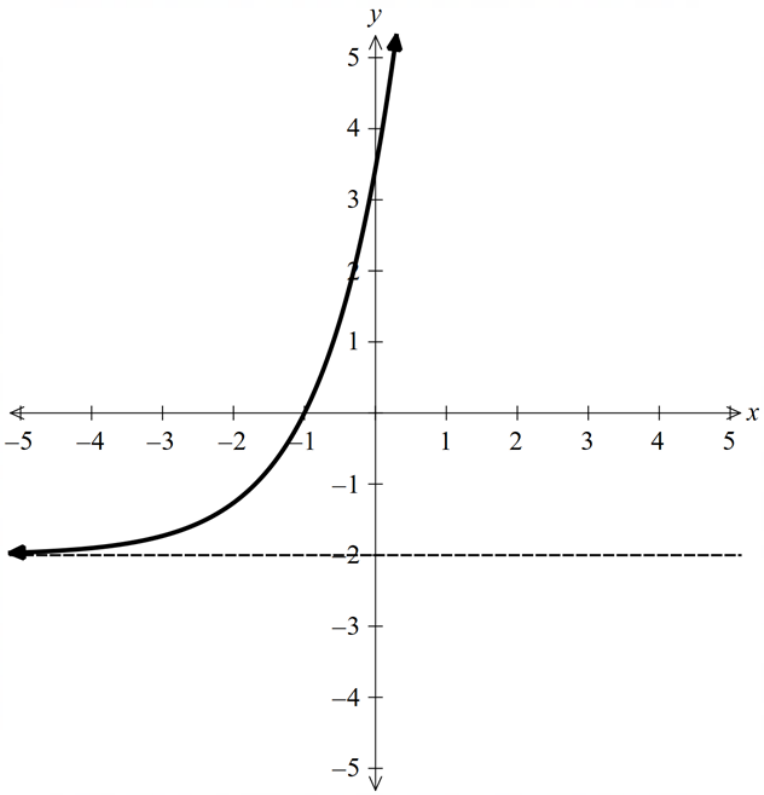
- Write your Teacher's Name and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 41 to 43 of Booklet 2. If you use this space, clearly indicate which question you are answering.

Q29 – Q32	/17
Q33 – Q35	/18

Please turn over

Question 29 (4 marks) **Marks**

The graph of $y = 2e^{x+b} + c$ is given below.



(a) In your own words, justify why $c = -2$. **1**

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(b) Show that $b = 1$. **1**

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Question 29 continues on page 31

Question 29 continued

(c) Find the area bounded by the curve, the $x-$ axis and the $y-$ axis. **2**

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Question 30 (3 marks)

When Lucio plays for his soccer team, they have an 85% chance of winning. When he does not play, their chance of winning is 40%.

The chance that Lucio plays any particular game is 90%.

(a) Find the probability that Lucio plays and his team wins. **1**

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(b) Find the probability that Lucio played, given that his team won. **2**

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Question 31 (4 marks) **Marks**

In a game of chance, a player is to spin a spinner with sectors of unequal sizes.

The following table gives the probability distribution of the money that the player could win.

Money prize	\$2000	\$4000	\$6000	\$8000
Probability	$\frac{2}{5}$	x	$\frac{1}{5}$	y

The expected value of the game is \$4000.

What is the probability of winning \$6000 or more? **4**

Question 32 (6 marks) **Marks**

An open topped fish tank of volume 30 m^3 is to be made in the shape of a rectangular prism with length $3x$ metres, width x metres and height h metres. Materials cost \$20 per square metre for the base of the tank and \$35 per square metre for the sides of the tank.

(a) Show that the total cost \$C of making the fish tank is given by **3**

$$C = 60x^2 + \frac{2800}{x}$$

Question 32 continues on page 35

Question 32 (continued)

(b) Hence find the dimensions of such a tank with the least total cost. Give your answer correct to 3 decimal places. **3**

Question 33 (7 marks) **Marks**

Consider the curve $y = 1 - e^{-x^2}$.

(a) Find any stationary points and determine their nature. **2**

(b) Show that there are points of inflection at $x = \pm\frac{1}{\sqrt{2}}$. **2**

Question 33 continues on page 37

Question 33 (continued) **Marks**

(c) What value does y approach when $x \to \infty$? 1

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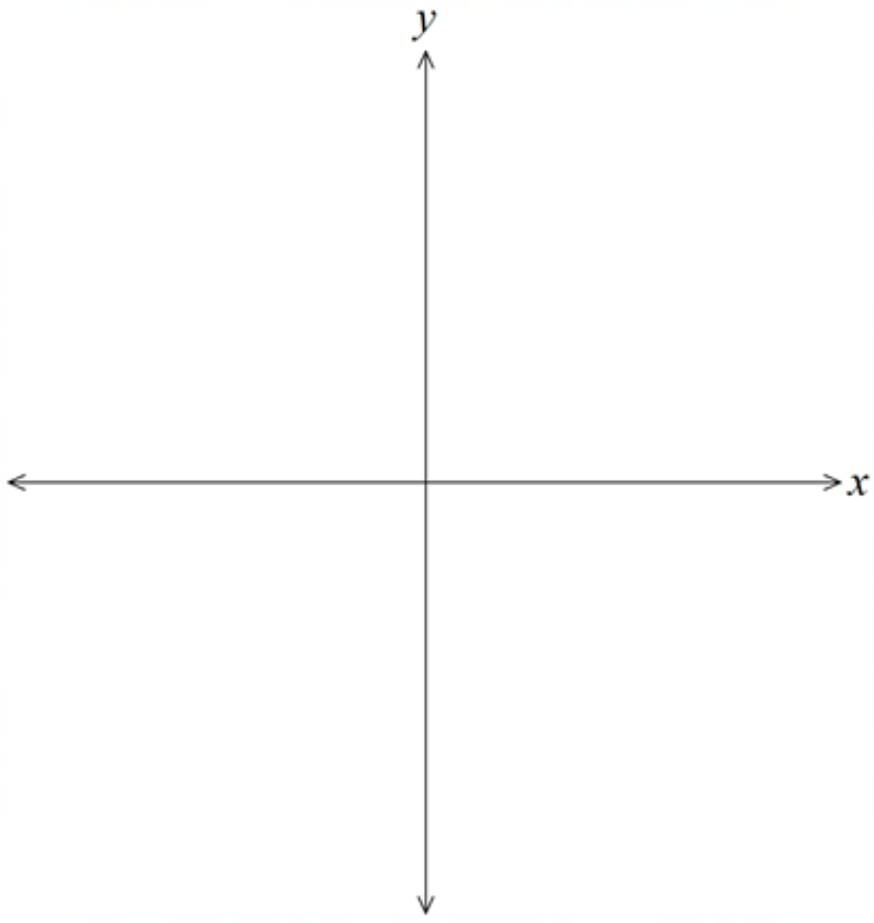
(d) Sketch $y = 1 - e^{-x^2}$, showing all asymptotes, turning points and points of inflection. **2**



Question 34 (7 marks) **Marks**

A circular Ferris wheel rotates at a constant rate, and the height of a certain passenger above the ground as a function of time can be modelled by the equation:

$$h(t) = 20\sin\left(\frac{\pi}{20}(t-15)\right) + 22,$$

where $h(t)$ represents the height in metres and t is the time in seconds.

(a) What is the maximum height of the person during the ride? **1**

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(b) Show that the Ferris wheel completes 15 full rotations during a 10-minute ride. **2**

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Question 34 continues on page 39

Question 34 (continued)

(c) A passenger can view the harbour from any height above 30m. After the Ferris wheel starts the ride, when will the passenger reach height of 30m for the first time? **2**

(d) How long will the passenger stay above 30m during the 10-minute ride? Answer in minutes and seconds. **2**

Question 35 (4 marks) **Marks**

A substance decomposes at a rate that is proportional to the mass of the substance remaining. The remaining mass (M) can be modelled using the formula $M = M_0 e^{-kt}$, where t is the time in hours, M_0 is the initial mass and k is a constant.

Initially there is 30 kg of the substance. After 24 hours half of the substance has decomposed.

Determine how long it will take for 90% of the substance to decompose. **4**

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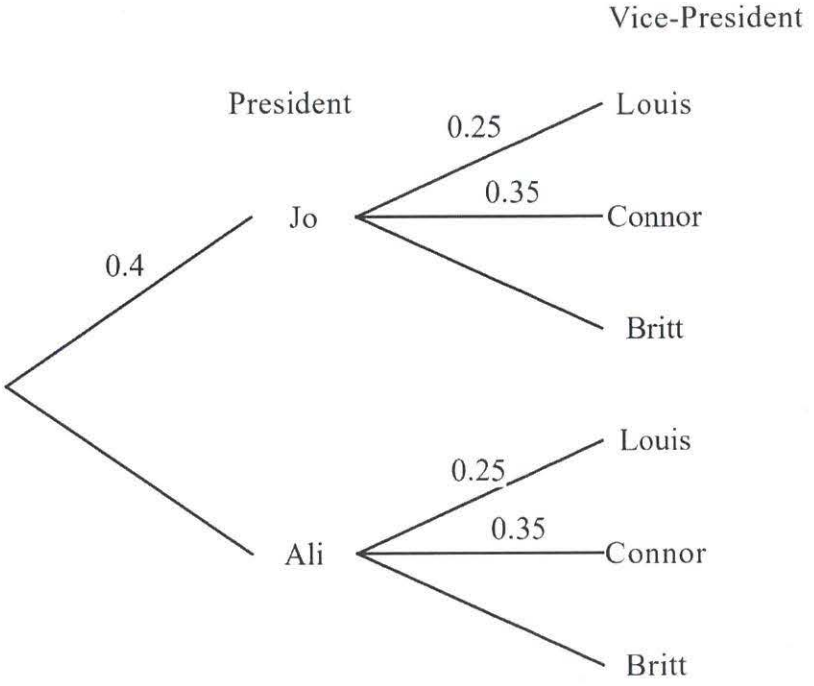
End of Examination

Section I

10 marks
Attempt Questions 1 – 10.
Allow about **20 minutes** for this section.
Use the **multiple-choice answer sheet** for questions 1 – 10.

1. The partially completed probability tree below shows the results of a survey of voting intentions of shareholders in an election for President and Vice-President of a large corporation.



Based on this survey, what is the probability that Jo is elected President, with either Britt or Louis as Vice-President?

(A) 0.04

(B) 0.26

(C) 0.30

(D) 0.50

$P(JB) + P(JL) = 0.4 \times 0.4 + 0.4 \times 0.25$
$= 0.4(0.4 + 0.25)$
$= 0.4 \times 0.65$
$= 0.26$

2. Which one of the following statements about the line $12x - 4y = 0$ is **NOT** true?

$-4y = -12x + 0$
$y = 3x$
$\therefore$ slope $= 3$

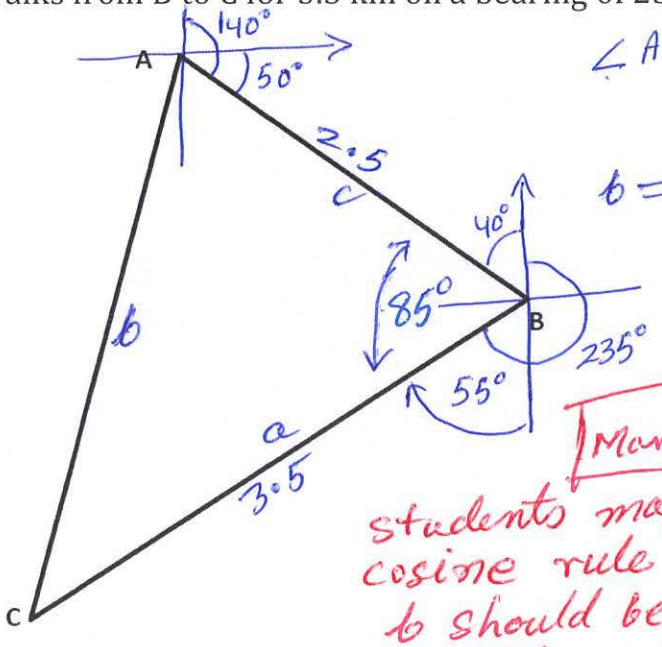
(A) The line has a slope of 12

(B) The line passes through the origin

(C) The line has the same slope as the line with the equation $12x - 4y = 12$

(D) For this line, as x increases y increases

3. On her bushwalk, Angela walks 2.5 km from A to B on a bearing of 140° and then walks from B to C for 3.5 km on a bearing of 235°.



$\angle ABC = 180° - 40° - 55°$
$= 85°$

$b = \sqrt{3.5^2 + 2.5^2 - 2 \times 3.5 \times 2.5 \cos 85°}$
$= 4.1$

Marker's Comment:
students may not need to use cosine rule at all. As $\angle ABC \doteq 85°$ b should be just less than $\sqrt{2.5^2 + 3.5^2}$ or less than 4.3

How far, to one decimal place, does Angela need to walk to return to the starting point?

(A) 5.2 km

(B) 4.4 km

(C) 4.1 km

(D) 3.1 km

4. The parabola $f(x) = -x^2 + 9mx + 2$ has a maximum at $x = \frac{2}{m}$, where m is a constant such that $m \neq 0$.

What are the values of m?

(A) $m = \frac{2}{3}$ or $m = -\frac{2}{3}$

(B) $m = \frac{2}{9}$ or $m = -\frac{2}{9}$

(C) $m = \frac{9}{4}$ or $m = -\frac{9}{4}$

(D) $m = \frac{3}{2}$ or $m = -\frac{3}{2}$

$f'(x) = -2x + 9m$
when $f'(x) = 0$ (for maxima)
$-2x + 9 = 0$
$2x = 9$
$x = \frac{9}{2}m$.
but $x = \frac{2}{m}$ (given)
$\therefore \frac{9}{2}m = \frac{2}{m}$
$9m^2 = 4$
$m = \frac{4}{9}$
$m = \pm\frac{2}{3}$

5. What are the coordinates of the turning point to the curve $y = e^x - ex$?

(A) $(0, 1)$

(B) $(e, 1)$

(C) $(1, e)$

(D) $(1, 0)$

$y' = e^x - e$
$e^x - e = 0$ (for turning point)
$e^x = e$
$x = 1$
$\therefore$ when $x = 1$, $y = e^1 - e \times 1$
$y = 0$
$\therefore (1, 0)$ is the point

6. A series has following terms:

2+5+8+11+14+17+ ...

What is the sum of the first 30 terms of the series

(A) 1410

(B) 1365

(C) 1380

(D) 2820

$d = 3, \quad a = 2$

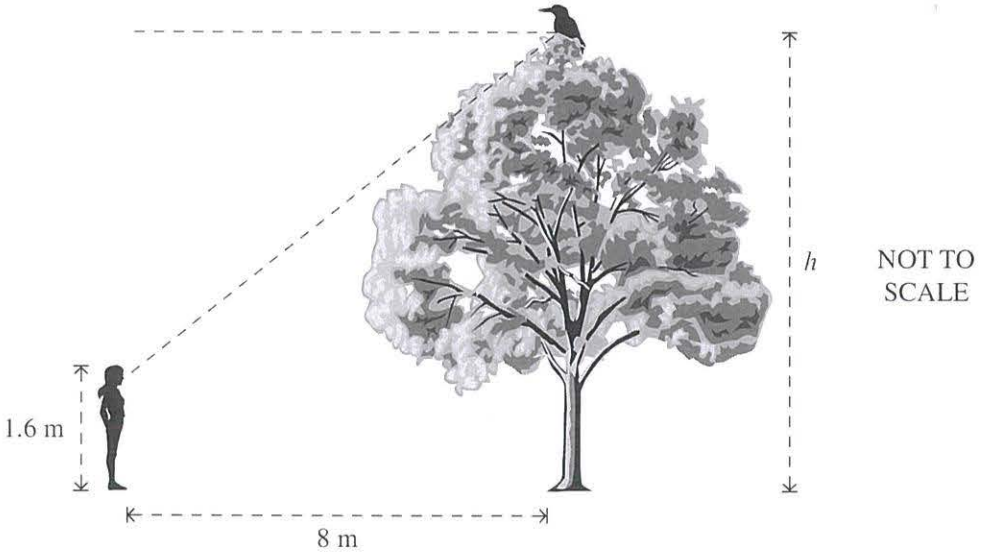
$S_{30} = \frac{n}{2}(2a + (n-1)d)$

$= \frac{30}{2}(2 \times 2 + (30-1) \times 3)$

$= 15 \times 91$

$= 1365$

7. Kelly is watching a bird at the top of a tree. Kelly is 1.6 m tall and is standing 8 m away from the tree, as shown in the diagram.



If the angle of depression of Kelly from the bird is 73°, what is the height of the tree?

(A) 2.45 m

(B) 4.05 m

(C) 26.17 m

(D) 27.77 m

$\tan 73^\circ = \frac{x}{8}$

$x = 8 \tan 73^\circ$

$= 26.17$

$+ 1.6$

27.77

8. The functions $g(x)$ and $h(x)$ are defined as follows:

$$g(x) = \frac{x^2}{8} \qquad h(x) = \sqrt{2x} + 3$$

Which is the correct expression for $h(g(x))$?

$h(g(x)) = \sqrt{2 \times \frac{x^2}{8}} + 3$

$= \sqrt{\frac{x^2}{4}} + 3$

$= \frac{x}{2} + 3$

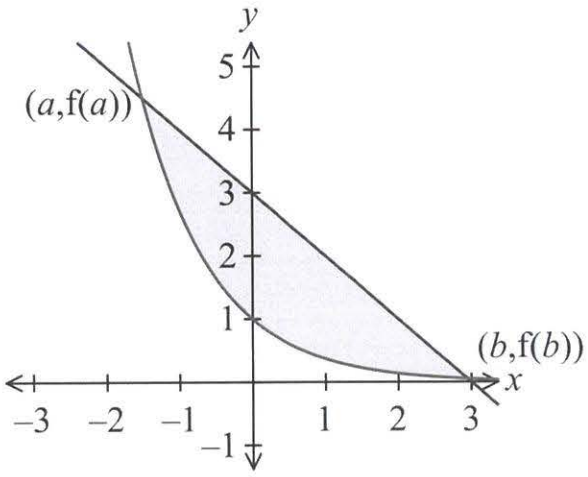
(A) $\sqrt{\frac{2x}{3}}$

(B) $\frac{x}{2} - 3$

(C) $\frac{x}{2} + 3$

(D) $\sqrt{\frac{x}{2} + 3}$

9. The diagram below shows the region enclosed by $x + y = 3$ and $y = e^{-x}$.



Which of the following integrals represents the area of the shaded region?

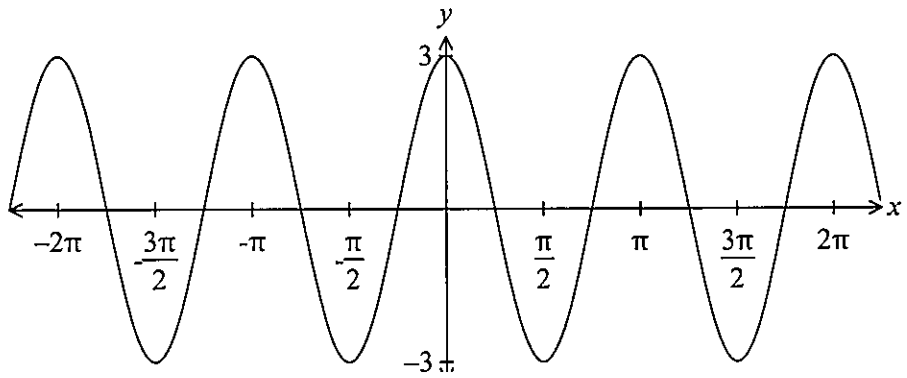
(A) $\int_a^b (e^{-x} - 3 + x)dx$

(B) $\int_a^b (3 - x + e^{-x})dx$

(C) $\int_a^b (e^{-x} + 3 + x)dx$

(D) $\int_a^b (3 - x - e^{-x})dx$

10. The graph of $y = f(x)$ is show below.



Which of the following could **NOT** be the correct description of $f(x)$?

(A) $y = 3\sin 2(x + \frac{\pi}{2})$

(B) $y = 3\sin(2x + \frac{\pi}{2})$

(C) $y = 3\cos 2x$

(D) $y = 3\cos 2(x - \pi)$

END OF SECTION I

12MAT AT4 2024 Q11-17 MARKER'S COMMENTS

(11)

$\log_3 5 = 2\log_3 10 - \log_3 x$

$\therefore \log_3 x = 2\log_3 10 - \log_3 5$

$= \log_3 10^2 - \log_3 5$

$= \log_3 \frac{100}{5}$

$\therefore x = \frac{100}{5}$

$\therefore x = 20$

2 marks	Complete solution.
1 mark	Correct use of a relevant log law that progresses toward a solution.

(12)

$x^2 + 8x + y^2 - 6y = 24$

$x^2 + 8x + 16 + y^2 - 6y + 9 = 24 + 16 + 9$

$(x+4)^2 + (y-3)^2 = 49$

$\therefore (x+4)^2 + (y-3)^2 = 7^2$

$\therefore$ centre is $(-4, 3)$, radius 7

2 marks	Complete solution
1½ marks	Correctly completes the square AND correctly finds the centre or the radius
1 mark	Correctly completes the square
or	Correctly finds the centre
or	Correctly finds the radius

12MAT AT4 2024 Q11-17 MARKER'S COMMENTS

(13) a)

5	8	13
7	4	11
12	12	24

b) $P(\text{Gym} \mid \text{Run}) = \frac{5}{13}$

This question was poorly attempted. More than 50% of students failed to read the question thoroughly enough to notice that this is a conditional probability question.
Also, those students using a formula to calculate P were wasting their time; for one mark, just read the answer from the table.

(14) a) $C = 10N + 300$

when $N = 20$, $C = 10(20) + 300$

$= \$500$

b) Straight line through (0, 300) and (40, 700)

2 marks Straight line through (0, 300) and (40, 700)

1 mark Mostly correct, but some error (not straight, doesn't make it to (40, 700) etc.)

12MAT AT4 2024 Q11–17 MARKER'S COMMENTS

c) Break even when Income = Costs

ie $30N = 10N + 300$

$20N = 300$

$N = 15$

Note that break **even** means where income = costs, not "at least" or "greater than".

d) when $N = 30$, Profit = Income − Costs

$= 30(30) - [10(30) + 300]$

$= 900 - 600$

$= \$300$ profit

15 a) $\angle B + 60° + 70° = 180°$ (angle sum of triangle)

$\therefore \angle B = 50°$

$\therefore \frac{AB}{\sin 60°} = \frac{8}{\sin 50°}$

$AB = \frac{8 \sin 60°}{\sin 50°}$

$= 9.044126$

$\doteq 9.04$ (2 decimal places)

2 marks Complete solution

1 mark Correctly shows why $\angle B = 50°$

or Otherwise correct solution, but missing reasoning for why $\angle B = 50°$

This is a "show" question – don't leave anything out (show everything!).

This is a two-mark question, and the answer is already given, so that can't be worth anything. Think about what the marks will be awarded for.

12MAT AT4 2024 Q11–17 MARKER'S COMMENTS

(15) b) $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 8 \times 9.04 \times \sin 70°$

$= 33.9792\ldots$

$\doteq 33.98\ cm^2$

OR

$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 8 \times \frac{8\sin 60°}{\sin 50°} \times \sin 70°$

$= 33.99479\ldots$

$\doteq 33.99\ cm^2$

(16) a) Inversely proportional, so $N = \frac{K}{A}$

when $N = 300$, $A = 1\ m^2$

$= (100 + 100)\ cm^2$

$= 10000$

$\therefore \quad 300 = \frac{k}{10000}$

$\therefore k = 3\,000\,000$

This question was very poorly done. Please revise inverse variation.

Note that obtaining the correct answer from an incorrect method did not attract any marks. Many students erroneously used direct variation.

12MAT AT4 2024 Q11–17 MARKER'S COMMENTS

(16) b) $N = \frac{3000000}{4500}$

$= 666.666...$

$\doteq 667$ people (nearest person)

(666 people was also accepted)

2 marks Complete solution

1 mark Correct substitution into correct formula.

This question was incredibly poorly done - please revise inverse variation!

No marks were awarded where students used models other than inverse variation, as this is such a fundamental part of the question.

Most common incorrect models used were exponential growth and direct variation, both of which attracted zero marks.

Note that $N = \frac{1}{K}A$ is <u>not</u> inverse variation.

12MAT AT4 2024 Q11-17 MARKER'S COMMENTS

(17) a) $\frac{T_2}{T_1} = \frac{-3(\ln x)^2}{\ln x}$

$= -3\ln x$

$\frac{T_3}{T_2} = \frac{9(\ln x)^3}{-3(\ln x)^2}$

$= -3\ln x$

$= \frac{T_2}{T_1}$

$\therefore$ the series is geometric.

This question was very poorly done. Students need to revise the method of proving a series is geometric (and proof more generally!).
Common errors which were awarded zero marks were:

- Assuming the result you were trying to prove. Proving two statements equal by starting with the assumption that they are equal (ie. LHS = RHS) will not earn you any marks.
- Assuming the sequence is geometric, then doing... anything really. Again, you can't assume what you have been asked to prove.
- Losing the minus, twice (ie $r = 3\ln x$). Getting it wrong twice doesn't make it right.

12MAT AT4 2024 Q11–17 **MARKER'S COMMENTS**

17 b) To have a limiting sum, $|r| < 1$

$$|-3\ln x| < 1$$

$$-1 < -3\ln x < 1$$

$$\tfrac{1}{3} > \ln x > -\tfrac{1}{3}$$

$$e^{\frac{1}{3}} > e^{\ln x} > e^{-\frac{1}{3}}$$

$$\therefore e^{-\frac{1}{3}} < x < e^{\frac{1}{3}}$$

2 marks Complete solution

1 mark Correctly identifies $|-3\ln x| < 1$

ok Correctly states $x < e^{\frac{1}{3}}$ or $x > e^{\frac{1}{3}}$

Read the question clearly: you are not asked to find the limiting sum. Any calculations about $\frac{a}{1-r}$ were irrelevant here.

MARKER'S COMMENTS - QUESTION (18)

Grassed area $= \frac{220}{2\times 2}[135 + 180 + 2(120)]$ using Trapezoidal Rule.

$= 55 \times 555$ — ① correct Trapezoidal Rule

$= 30525\ m^2$

Area of whole park = Area of rectangle + Area of semi-circle

Finding area of whole park (or equivalent ment) ① — $= 180 \times 220 + \frac{1}{2} \times \pi \times 110^2$

$= 39600 + 19006.6355...$

$= 58606.6355...\ m^2$

∴ Area of lake = Area of whole park − Grassed area

$= 58606.635... - 30525$

$= 28081.635...$

$\approx 28082\ m^2$ — ① correct area of lake

- Students need to revise their work on Trapezoidal Rule. Make sure you know how the formula works (if you're going to use it). otherwise, just do the area of a trapezium!

- "Two applications of the Trapezoidal Rule" just means split the area into two trapezia, not use the Rule twice on different sections.

- Make sure you familiarise yourself with the phrases: "sub intervals", "function values", "number of applications" and what they mean.

MARKER'S COMMENTS - QUESTION (19)

(a) $T_3 = ar^2$ $\quad T_6 = ar^5$

$18 = ar^2$ $\quad 486 = ar^5$

$\therefore a = \frac{18}{r^2}$ —① $\quad \therefore a = \frac{486}{r^5}$ —②

Sub ① into ②: $\frac{18}{r^2} = \frac{486}{r^5}$

$18r^5 = 486r^2$

$r^3 = \frac{486}{18}$

$r^3 = 27$

$\therefore r = 3$ —① correct common ratio

$\therefore a = \frac{18}{3^2}$

$= 2$ —① correct first term (CFPE)

- A reminder that if $x^3 = 27$, then $x \neq \pm 3$. If $x = -3$, $x^3 = (-3)^3 = -27$.

(b) $S_{10} = \frac{a(r^{10}-1)}{r-1}$

$= \frac{2(3^{10}-1)}{3-1}$ (CFPE allowed from previous part)

$= 3^{10} - 1$

$= 59048$

- Students need to revise their APs and GPs formulae!

MARKER'S COMMENTS - QUESTION (20)

$y = 3\ln 2x$

$\frac{dy}{dx} = 3 \times \frac{2}{2x}$

$= 3 \times \frac{1}{x}$

$= \frac{3}{x}$ — (1) correct derivative

when $x = 2$, $y = 3\ln 4$ $\therefore P = (2, 3\ln 4)$ — ($\frac{1}{2}$) finding y-coordinate

and $\frac{dy}{dx} = \frac{3}{2}$

so gradient of tangent $= \frac{3}{2}$

$\therefore$ gradient of normal $= -\frac{2}{3}$ — (1) correct gradient at $x = 2$

$\therefore$ equation of normal: $y - 3\ln 4 = -\frac{2}{3}(x - 2)$

— ($\frac{1}{2}$) correct substitution into formula. (CFPE allowed)

- This question was mostly done well, however, lots of silly mistakes — e.g. $3 \times \frac{1}{x} = 3x$

- Some students need to revise differentiation rules!

MARKER'S COMMENTS - QUESTION (21)

$$\int_1^3 (f(x)+x)\,dx = \int_1^3 f(x)\,dx + \int_1^3 x\,dx$$

$$= \int_1^6 f(x)\,dx - \int_3^6 f(x)\,dx + \left[\frac{x^2}{2}\right]_1^3$$

$$= 6 - -4 + \left(\frac{9}{2} - \frac{1}{2}\right)$$

$$= 10 + \frac{8}{2}$$

$$= 10 + 4$$

$$= 14$$

(1) mark: $6 - -4 = 6 + 4$

$= 10$ [for $f(x)$]

(1) mark: evaluating $\left[\frac{x^2}{2}\right]_1^3 = 4$ and correct final solution (CFPE allowed)

- This question was done well by most students.

- Some students found it helpful to draw a diagram of the areas above and below the curve. to help visualise.

(22) $y = (1+e^{2x})^8$

$$\frac{dy}{dx} = 8(1+e^{2x})^7 \times 2e^{2x}$$

$$= 16e^{2x}(1+e^{2x})^7$$

(1) mark: correct derivative of $[f(x)]^8$.

(1) mark: correct derivative of $1+e^{2x}$.

- This question was done well!

MARKER'S COMMENTS - QUESTION (23)

$x(t) = 12 - 2t + 12\log_e(t+3)$

(a) $x(0) = 12 - 2(0) + 12\log_e(0+3)$

$= 12 + 12\log_e 3$ m to the right of the origin.

(OR 25.2 m)

(1) mark

- Done well!

(b) $v = -2 + 12 \times \frac{1}{t+3}$ — (1) correct derivative

$= -2 + \frac{12}{t+3}$

At rest when $v = 0$,

$0 = -2 + \frac{12}{t+3}$

$2 = \frac{12}{t+3}$

$t+3 = \frac{12}{2}$

$t+3 = 6$

$\therefore t = 3$ — (1) solving for t when $v=0$ OR substituting $t=3$ into derivative to show $v=0$.

- Students who solved the equation $v=0$ would have found that there is only one turning point which is useful for part (c).

MARKER'S COMMENTS - QUESTION (23)

(c) Since there is only one turning point (at $t=3$), the object only changes direction <u>once</u> for all t.

① mark: showing that $v<0$ when $t>3$.

Test $t=3$:

t	2	3	4
v	$\frac{2}{5}$	0	$-\frac{2}{7}$
slope	/	—	\

① mark: the object is moving to the left / backwards / towards the origin

$\therefore$ There is a maximum turning point when $t=3$, so for $t>3$, the object is moving to the left since velocity is negative.

- This question was poorly answered. Majority of students were not able to identify/explain the link between when $v=0$ and when the <u>object</u> changes direction.

- <u>Many</u> students incorrectly stated that because the acceleration is negative, that means the <u>object</u> is travelling to the left. This is wrong!!!

MARKER'S COMMENTS - QUESTION (23)

ALTERNATIVE SOLUTION:

(c) $t > 3$

$t + 3 > 6$

$\frac{1}{t+3} < \frac{1}{6}$

$\frac{12}{t+3} < \frac{12}{6}$

$\frac{12}{t+3} < 2$

$-2 + \frac{12}{t+3} < 2 - 2$

$-2 + \frac{12}{t+3} < 0$

$\therefore V < 0$ when $t > 3$. (2) marks: full correct solution.

MARKER'S COMMENTS

2024 Advanced Trial Solutions (Q24 → Q28)

24a) Show $\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A} = \sec^2 A$

Method 1

LHS $= \dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A}$

$= \dfrac{\frac{1}{\sin A}}{\frac{1}{\sin A} - \sin A}$ } 1 mk

$= \dfrac{\frac{1}{\sin A}}{\frac{1 - \sin^2 A}{\sin A}}$

$= \dfrac{1}{\sin A} \times \dfrac{\sin A}{1 - \sin^2 A}$

$= \dfrac{1}{1 - \sin^2 A}$ $(\sin^2 A + \cos^2 A = 1)$ $-\frac{1}{2}$ mk

$= \dfrac{1}{\cos^2 A}$ $-\frac{1}{2}$ mk

$= \dfrac{1}{\sec^2 A}$

$=$ RHS

MARKER'S COMMENTS

Method 2

$$LHS = \frac{\text{cosec } A}{\text{cosec } A - \sin A}$$

$$= \frac{\frac{1}{\sin A}}{\frac{1}{\sin A} - \sin A} \times \frac{\sin A}{\sin A} \quad \} \text{ 1 mk}$$

$$= \frac{1}{1 - \sin^2 A} \quad - \tfrac{1}{2} \text{ mk}$$

$$= \frac{1}{\cos^2 A} \quad - \tfrac{1}{2} \text{ mk}$$

$$= \sec^2 A$$

$$= RHS$$

MARKER'S COMMENTS

Method 3

$$\text{LHS} = \frac{\text{cosec } A}{\text{cosec } A - \sin A}$$

$$= \frac{1}{\sin A} \div (\text{cosec } A - \sin A)$$

$$= \frac{1}{\sin A} \times \frac{1}{\text{cosec } A - \sin A}$$

$$= \frac{1}{\sin A \ (\text{cosec } A - \sin A)}$$

$$= \frac{1}{\sin A \times \text{cosec } A - \sin^2 A} \quad \text{— 1 mk}$$

$$= \frac{1}{\sin A \times \frac{1}{\sin A} - \sin^2 A}$$

$$= \frac{1}{1 - \sin^2 A} \quad (\sin^2 A + \cos^2 A = 1) \quad \text{— ½ mk}$$

$$= \frac{1}{\cos^2 A} \quad \text{— ½ mk}$$

$$= \sec^2 A$$

Marker's Comment

Quite well done.

However, a few students thought that the reciprocal of $\frac{1}{\sin A} - \sin A$ is $\sin A - \frac{1}{\sin A}$ which is not true.

Also, remember that $1 + \tan^2 x = \sec^2 x$

MARKER'S COMMENTS

24b) $\int \tan^2 x \left(\frac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \right) dx$

$\int \tan^2 x \sec^2 x \, dx$ $-\frac{1}{2}$ mk

$\int \sec^2 x (\tan x)^2 dx$

$\int f'(x) [f(x)]^n dx$

$= \frac{(f(x))^{n+1}}{n+1} + C$

$= \frac{\tan^3 x}{3} + C$ $-\frac{1}{2}$ mk

Marker's Comments

Some students were not able to recognise that this question can be done using the concept of reverse chain rule. A few students used integration by substitution.

MARKER'S COMMENTS

Q25 $\int_1^3 \frac{3x^2 - 1}{5x^3 - 5x + 9}\, dx$

$= \frac{1}{5} \int_1^3 \frac{5(3x^2 - 1)}{5x^3 - 5x + 9}\, dx$

This is in the form of $\int \frac{f'(x)}{f(x)}\, dx$

$= \ln|f(x)| + c$

$= \frac{1}{5} \ln\left[\left|5x^3 - 5x + 9\right|\right]_1^3$ — 1 mk

$= \frac{1}{5}\left[\ln\left|5(3)^3 - 5(3) + 9\right| - \ln\left|5(1)^3 - 5(1) + 9\right|\right]$

$= \frac{1}{5}\left[\ln\left|135 - 15 + 9\right| - \ln\left|5 - 5 + 9\right|\right]$

$= \frac{1}{5}\left[\ln|129| - \ln|9|\right]$ — 1 mk

$= \frac{1}{5} \ln\left|\frac{129}{9}\right|$ } or 1 mk

$= \frac{1}{5} \ln\left|\frac{43}{3}\right|$

Marker's Comments

Quite well done. A number of students forgot to include the absolute value sign. Also, a number of students left the answer as $\frac{1}{5}\left[\ln|129| - \ln|9|\right]$ which is not not the most simplified form.

MARKER'S COMMENTS

A few students used integration by substitution to arrive at the correct answer.

Method 1

26) $\sec^2\theta = 2 \qquad 0 \le \theta \le \pi$.

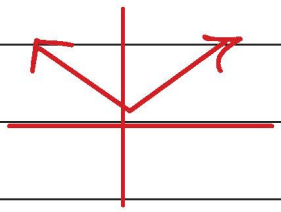
$\therefore \dfrac{1}{\cos^2\theta} = 2$

$\cos^2\theta = \dfrac{1}{2}$

$\cos\theta = \pm\dfrac{1}{\sqrt{2}}$ — 1 mk

$\theta = \pi/4$ (Related angle)

$\therefore \theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}$ — $\frac{1}{2}$ mk, $\frac{1}{2}$ mk



Method 2

$\sec^2\theta = 2$

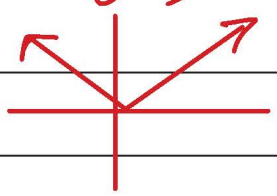
$1 + \tan^2\theta = 2$

$\tan^2\theta = 1$

$\tan\theta = \pm 1$ — 1 mk

$\theta = \pi/4$ (related angle)

$\therefore \dfrac{\pi}{4}, \dfrac{3\pi}{4}$ — $\frac{1}{2}$ mk, $\frac{1}{2}$ mk



Marker's comments

Generally well done.

½ a mark was deducted for extra solutions.

½ a mark was deducted for leaving the answer in degrees.

MARKER'S COMMENTS

Q27

$f(x) = (x+2)^2 - 1$

x-intercept (Let $f(x) = 0$)

$0 = (x+2)^2 - 1$

$(x+2)^2 = 1$

$x+2 = \pm 1$

$x+2 = 1$ or $x+2 = -1$

$x = -1$ $\quad$ $x = -3$

∴ x-intercepts are $(-1, 0)$ and $(-3, 0)$

y-intercept (Let $x = 0$)

$f(0) = (0+2)^2 - 1$

$= 4 - 1$

$= 3$

∴ y-intercept = $(0, 3)$.

Concavity is concave up.

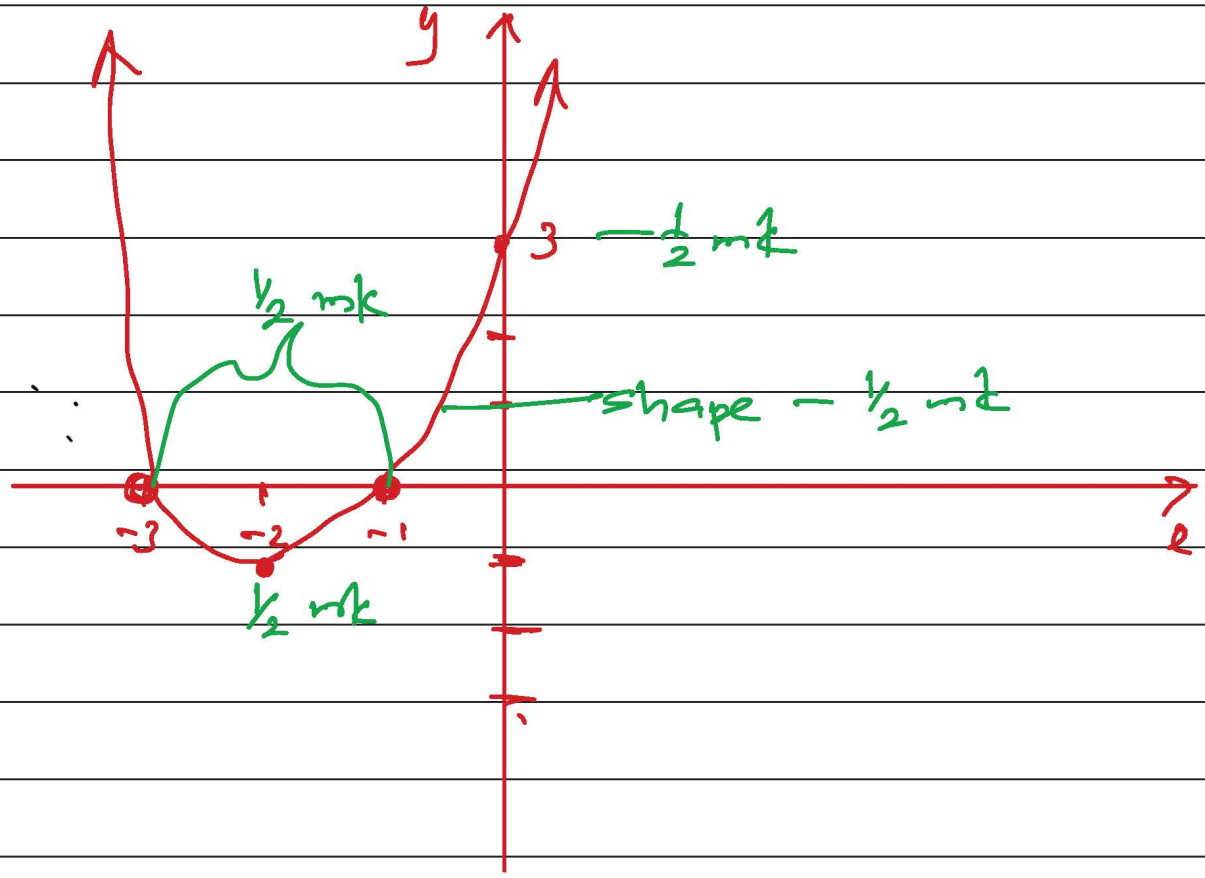
Turning point

Add the 2 x-intercepts and then divide by 2. $\frac{-1-3}{2} = -2$ or use $x = \frac{-b}{2a}$

y-coordinate of the turning point is:

$f(-2) = (-2+2)^2 - 1 = -1$ ∴ $(-2, -1)$

MARKER'S COMMENTS



Marker's Comments

This question was quite well done. Recall that numbers on the x and y-axis should be evenly spread out, according to an appropriate scale.

MARKER'S COMMENTS

27b) Could have been attempted using 2 different methods: one which involves factorisation and the second method which does not involve factorisation.

Using factorisation

$$g(x) = 2f(3-x)$$
$$= 2f(-(-3+x))$$
$$= 2f(-(x-3))$$

∴ the transformations involved in the correct order could have been:

1) vertical dilation by a scale factor of 2. — ½ mk — ½ mk
2) Reflection in the y-axis or horizontal dilation by a factor of −1.
3) Horizontal translation of 3 units to the right. — ½ mk — ½ mk

½ mk was awarded for the correct order of transformations

MARKER'S COMMENTS

For the factorised form, there are 2 more options:

1) Horizontal dilation by a factor of -1.
2) Vertical dilation by a scale factor of 2.
3. Horizontal translation of 3 units to the right.

For the factorised form:

1) Horizontal dilation by a factor of -1.
2) Horizontal translation of 3 units to the right.
3) Vertical dilation by a scale factor of 2.

MARKER'S COMMENTS

You could have checked whether your transformations are in the correct order by graphical or algebraic methods.

Algebraic Method

1) Horizontal dilation with scale factor of -1. (HD)

Replace x by $\frac{x}{-1} = -x$

$y = f(x)$

$y = f(-x)$

2) Vertical dilation by scale factor of 2. (VD)

Replace y by $y/2$

$y = f(-x)$ → from (1)

$y/2 = f(-x)$

$\therefore y = 2f(-x)$

3) Horizontal translation of 3 (HT) units to the right (replace x by $x-3$)

$y = 2f(-x)$ – from (2)

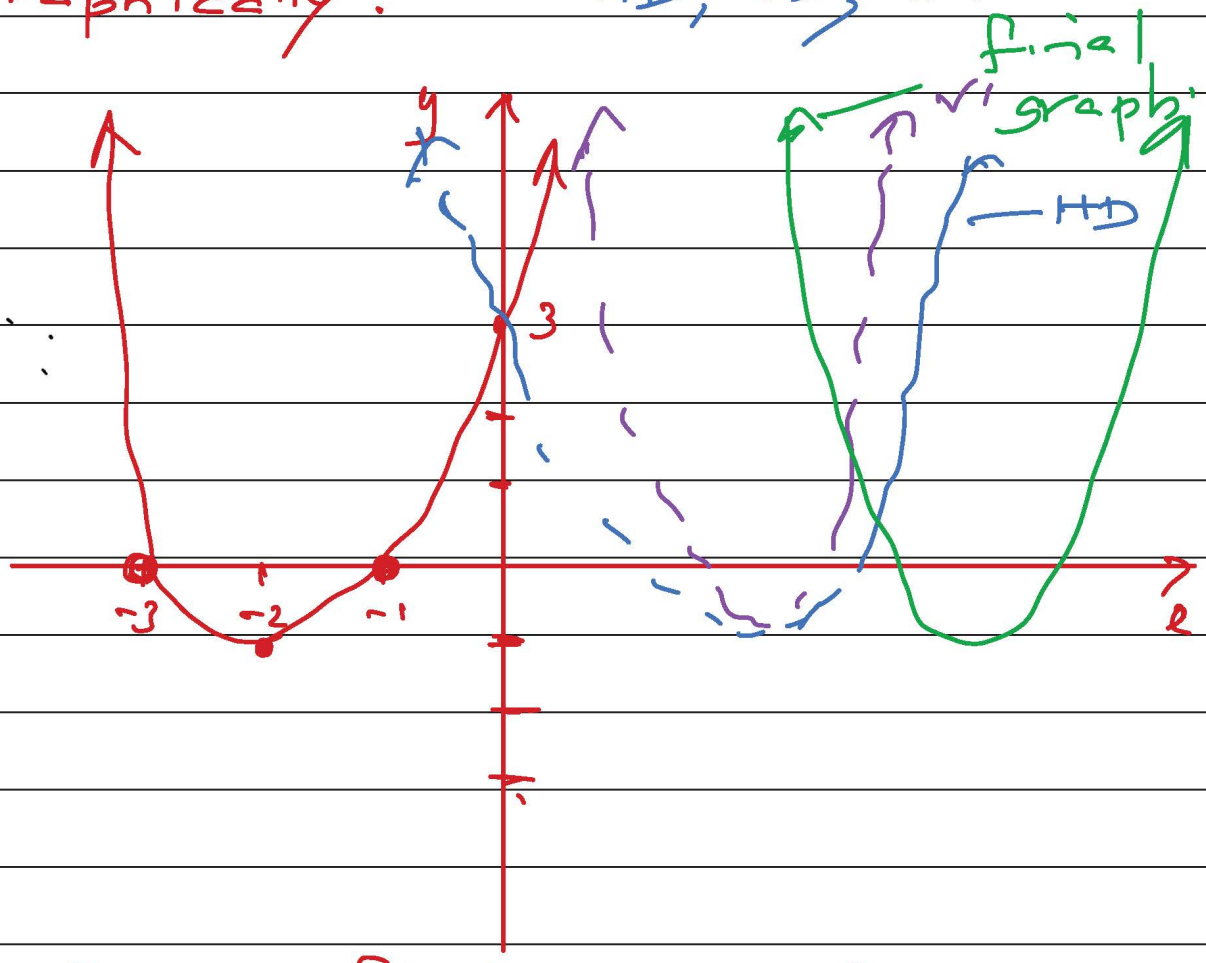
$= 2f(-(x-3))$

$= 2f(-x+3)$

$= 2f(3-x)$ → original question.

MARKER'S COMMENTS

Graphically : HD, VS, HT



If your final graph does not land at the same spot as the green graph, then your order of transformations is incorrect.

MARKER'S COMMENTS

The unfactorised form involved the following transformations.

1) Horizontal translation of 3 units to the right
2) Horizontal dilation by a factor of -1
3) Vertical dilation by a scale factor of 2.

Summary of the order of transformations

HD	HD
VD	HT
HT	VD
VD	HT
HD	HD
HT	VD
HT	VD
VD	HT
HD	HD

Marker's Comments

This question was poorly attempted. More remedial work is required.

- factorised form
- unfactorised form

MARKER'S COMMENTS

Q28.

$\theta = 36°$ or $\frac{2\pi}{10} = \pi/5$ is the — ½ mk

decagon.

Area $= 10 \times \frac{1}{2} \times r \times r \times \sin \frac{\pi}{5}$ — ½ mk

$100 = 5r^2 \times 0.587785252\ldots$

$r^2 = 34.026\,03233\ldots$

$r = 5.833\,183722\ldots$

≈ 5.83 — 1 mk

b) $AB^2 = 5.83^2 + 5.83^2 - 2 \times 5.83 \times 5.83 \times \cos \pi/5$ — ½ mk

$AB^2 = 12.99\,678785\ldots$

$\therefore AB = 3.605\,105\,803\ldots$ — ½ mk × 10

$= 36.05\,105\,803\ldots$ — ½ mk

≈ 36.1 (3 s.f.) (½ mk)

Markers' Comments — Quite well done.
If a wrong formula was used, no marks were awarded.
Students should use the following guideline: $r^2 = \_\_\_$

$r = \pm \sqrt{\_\_\_}$

$\therefore r = \_\_\_$ (as $r > 0$)

MARKER'S COMMENTS - QUESTION

Question 29

(a) $y = 2e^{x+b}$ has horizontal asymtote at $y=0$

since the asymtote of $y = 2e^{x+b}+c$ is $y=-2$, the graph has been translated 2 units down and c represents vertical translation.

$\therefore c = -2$ — (1)

Marker's comments

Most students got (1) mark.

Some students received ½ mark if they talked of translation of 2 units down but did not mention 'asymtote'.

(b) When $x=-1$, $y=0$

$2e^{-1+b} - 2 = 0$ (from part (a))

$2e^{-1+b} = 2$

$e^{-1+b} = 1$

$\ln e^{-1+b} = \ln 1$

$-1+b = 0$

$\therefore b = 1$ — (1)

Marker's Comment

only a few students lost mark in this

MARKER'S COMMENTS - QUESTION

Question 29

(c) $A = \int_{-1}^{0} (2e^{x+1} - 2)\, dx$

$= [2e^{x+1} - 2x]_{-1}^{0}$ — ①

$= (2e^{1} - 0) - (2e^{0} - 2x{-}1)$

$= 2e - (2 - -2)$

$= 2e - 4$ u^2 — ①

Marker's comment:

(1) mark was given for correct integration + correct boundaries.

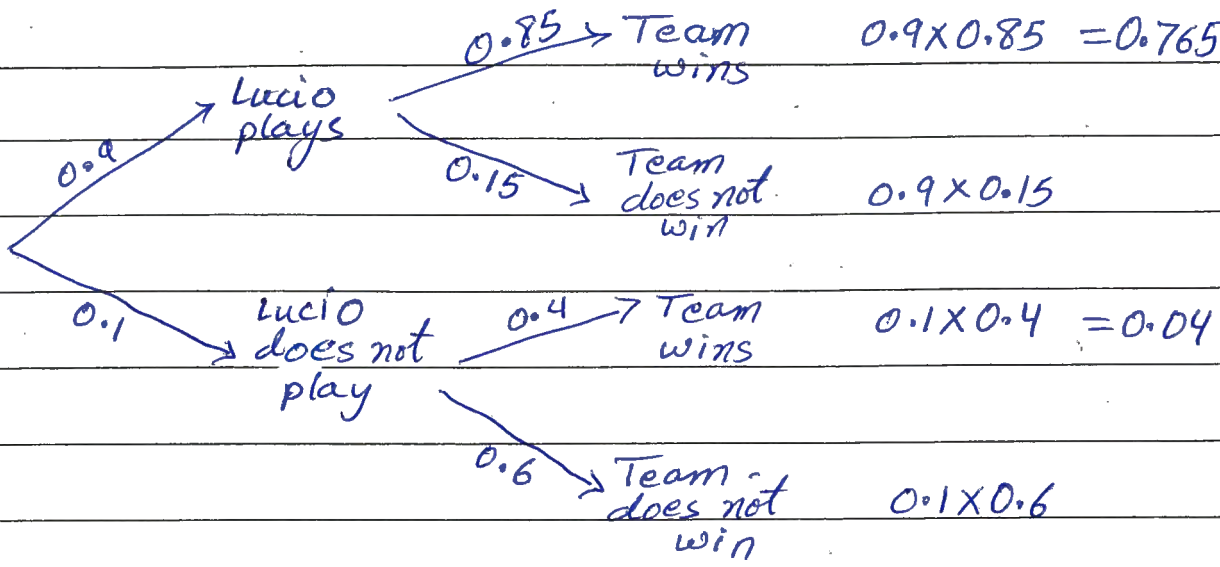
If integration was correct but boundary/boundries were wrong but rest of the working was correct then (1) mark given.

Few students tried to integrate $\int x\, dy$, and nearly all who tried did not succeed in getting correct answer. Either they could not integrate correctly or they had wrong boundaries.

MARKER'S COMMENTS - QUESTION

Question 30:

(a)



P (Lucio plays & team wins) = 0.765 — ①

or $\frac{153}{200}$

Marker's Comment:

Most people got this right. Those who got this wrong, may be had difficulty in reading and did not know how to draw a tree diagram.

(b) $P(\text{Lucio Played} \mid \text{Team Won}) = \frac{0.765}{0.765 + 0.04}$ → ① → ①

$= 0.95$ or $\frac{153}{161}$

① mark was easy if they got the numerator right. Many did not get the denominator right. Large number of students received only 1 mark in this question.

MARKER'S COMMENTS - QUESTION

Question 31

$x+y = 1-\left(\frac{2}{5}+\frac{1}{5}\right)$ — ①

$x+y = \frac{2}{5}$ — ①

$2000\times\frac{2}{5} + 4000x + 6000\times\frac{1}{5} + 8000y = 4000$

$800 + 4000x + 1200 + 8000y = 4000$

$4000x + 8000y = 2000$

$2x + 4y = 1$ — ② — ①

② − 2×①

$2x + 4y = 1$

$-\quad 2x + 2y = \frac{4}{5}$

$2y = \frac{1}{5}$

$y = \frac{1}{10}$ — ①

$P(\text{win} \geq 6000) = \frac{1}{5} + \frac{1}{10}$

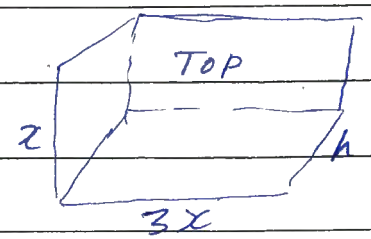
$= \frac{3}{10}$ — ①

Marker's comment

Most students got this question right. Only a few student made minor errors & some did not go beyond finding the values of x & y.

MARKER'S COMMENTS - QUESTION

Question 32



(a) $V = 3x \times x \times h$

$30_{m^3} = 3x^2 h$

$h = \frac{30}{3x^2}$

$h = \frac{10}{x^2}$ — (1) — (1)

cost of the base $= 3x^2 \times 20$

$= \$60x^2$

cost of 4 sides $= [2(3x \times h) + 2 \times h] \times 35$

$= 8 \times h \times 35$

$= \$280 \times h$

Total cost $= 60x^2 + 280 \times h$ — (1)

(2)

(1) into (2)

$= 60x^2 + 280 \times \frac{10}{x^2}$

$= 60x^2 + \frac{2800}{x}$ — (1)

Marker's Comment:

Only a few students lost any mark in this question. If a student could get $h = \frac{10}{x^2}$, they were able to answer the rest correctly.

MARKER'S COMMENTS - QUESTION

Question 32

(b) $C = 60x^2 + \frac{2800}{x}$

$= 60x^2 + 2800x^{-1}$

$\frac{dC}{dx} = 120x - 2800x^{-2}$

$= 120x - \frac{2800}{x^2}$ —(1)

for maximum/minimum point $\frac{dC}{dx} = 0$

$\therefore 120x - \frac{2800}{x^2} = 0$

$120x = \frac{2800}{x^2}$

$120x^3 = 2800$

$x = \sqrt[3]{\frac{2800}{120}}$

$= \sqrt[3]{70/3}$

$\doteqdot 2.8575$

$\frac{d^2C}{dx^2} = 120 + \frac{5600}{x^3}$

as $\frac{d^2C}{dx^2} > 0$ for all values of x, the curve is concave up around $x = \sqrt[3]{\frac{70}{3}}$ —(1)

$\therefore$ dimensions are $\sqrt[3]{70/3}$, $3x\sqrt[3]{\frac{70}{3}}$, $\frac{10}{(\sqrt[3]{70/3})^2}$ —(1)

$= 2.858_m, 8.574_m, 1.225_m$.

Marker's Comment:

No mark taken away for leaving the answers in exact form.

(1) mark taken away for not showing how it is a minimum Point.

(½) mark taken away if values were not put in the table when showing concavity.

Question 33

MARKER'S COMMENTS

a) $y = 1 - e^{-x^2}$

$\frac{dy}{dx} = -e^{-x^2} \times -2x$

$\frac{dy}{dx} = 2xe^{-x^2}$ $-\frac{1}{2}$ for differentiating

For stationary points, let $\frac{dy}{dx} = 0$.

$2xe^{-x^2} = 0$

$2x = 0$, but e^{-x^2} cannot equal zero

$\therefore x = 0$

When $x = 0$, $y = 1 - e^{-0^2} = 1 - 1 = 0$

$\therefore$ Stationary point at $(0, 0)$ $-\frac{1}{2}$ for whole point, including y value

Test nature: (1 mark in total for this)

Option 1:

x	-1	0	1
$\frac{dy}{dx}$	$-\frac{2}{e} = -7.36$	0	$\frac{2}{e} = +7.36$
Slope	\	—	/

$-\frac{1}{2}$ for table with correct values included.

$\therefore$ There is a minimum turning point at $(0,0)$. $-\frac{1}{2}$ mark.

Question 33 MARKER'S COMMENTS

part a) continued.

Test nature:

Option 2: If $\frac{dy}{dx} = 2xe^{-x^2}$

let $u = 2x \quad v = e^{-x^2}$

$u' = 2 \quad v' = -2xe^{-x^2}$

$$\frac{d^2y}{dx^2} = uv' + vu'$$
$$= 2x \times -2xe^{-x^2} + e^{-x^2} \times 2$$
$$= 2e^{-x^2} - 4x^2e^{-x^2}$$
$$= 2e^{-x^2}(1-2x^2)$$

When $x=0$, $\frac{d^2y}{dx^2} = 2e^{-0^2}(1-2(0)^2)$
$= 2 \times 1 \times (1-0) = 2$] $\frac{1}{2}$ mark

$\therefore \frac{d^2y}{dx^2} > 0$

$\therefore$ the curve is concave up.

$\therefore$ There is a minimum turning point at $(0,0)$.

$-\frac{1}{2}$ mark.

MARKER'S COMMENTS

Question 33

b) If $\frac{dy}{dx} = 2xe^{-x^2}$

Let $u = 2x \quad v = e^{-x^2}$

$u' = 2 \quad v' = -2xe^{-x^2}$

$\frac{d^2y}{dx^2} = uv' + vu'$

$= 2x \times -2xe^{-x^2} + e^{-x^2} \times 2$ ← ½ mark

$= 2e^{-x^2} - 4x^2e^{-x^2}$

$= 2e^{-x^2}(1-2x^2)$

Many students did not complete product rule to find 2nd derivative.

To find possible points of inflection, let $\frac{d^2y}{dx^2} = 0$

$2e^{-x^2}(1-2x^2) = 0$

$2e^{-x^2} \neq 0 \qquad 1-2x^2 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}}$

½ mark (CFPE allowed)

x	-1	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	1
$\frac{d^2y}{dx^2}$	$-2/e$ $= -0.74$	0	2	0	$-2/e$ $= -0.74$
concavity	$\cap$	•	$\cup$	•	$\cap$

½ mark - must show values

Since there is a concavity change either side of $x = -\frac{1}{\sqrt{2}}$ and $x = \frac{1}{\sqrt{2}}$, there are points of inflection at $x = \pm\frac{1}{\sqrt{2}}$.

½ mark to explain change in concavity.

Question 33

MARKER'S COMMENTS

c) When $x \to \infty$, $x^2 \to \infty$

$-x^2 \to -\infty$

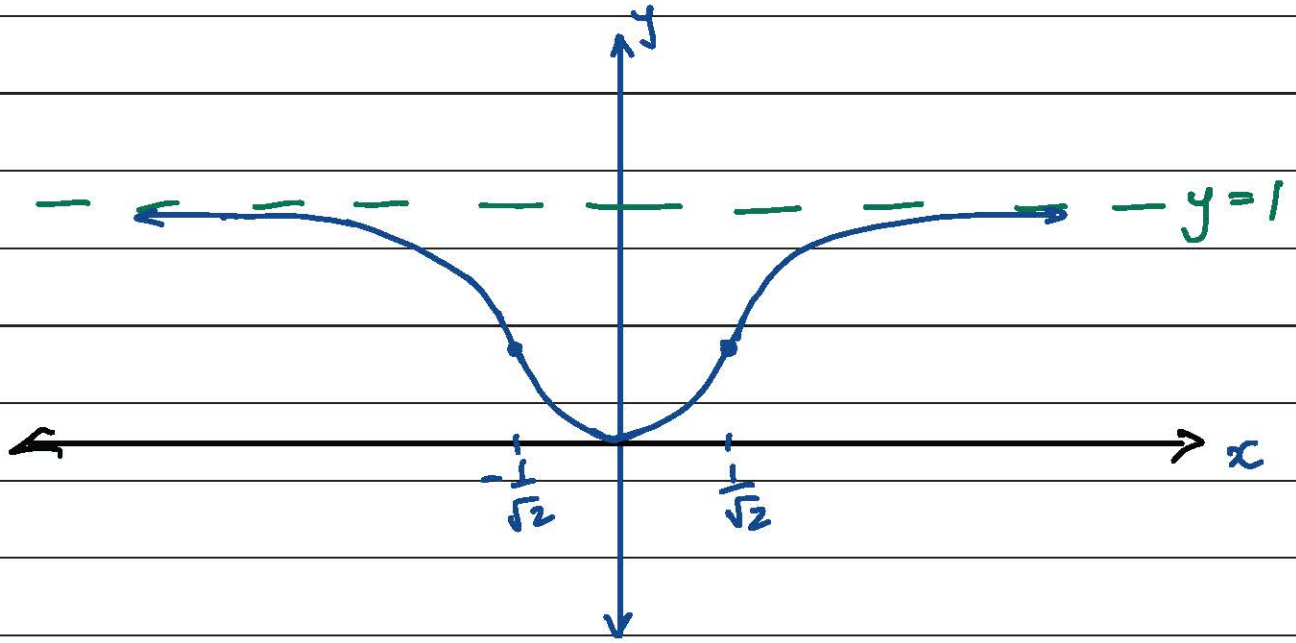
$e^{-x^2} \to 0$

$1 - e^{-x^2} \to 1$

$\therefore$ When $x \to \infty$, $y \to 1$. — 1 mark.

d) When $x \to -\infty$, $x^2 \to \infty$

$1 - e^{-x^2} \to 1$



$\frac{1}{2}$ mark - for minimum turning point at (0,0) with correct shape.

$\frac{1}{2}$ mark - for both points of inflection with correct shape

1 mark - for curve approaching asymptote correctly on both sides (lose $\frac{1}{2}$ mark if curve is not close enough to $y = 1$ or if it turns away).

Question 34 MARKER'S COMMENTS

a) A basic sine curve $\sin t$ has range $[-1, 1]$.

$\therefore$ The maximum value is 1.

$\therefore$ The sine curve $20\sin\left(\frac{\pi}{20}(t-15)\right)$ has a maximum value of $20 \times 1 = 20$

$\therefore$ $20\sin\left(\frac{\pi}{20}(t-15)\right) + 22$ has a maximum value of $20 + 22 = \underline{42\text{ m}}$ — 1 mark

b) Period $= \frac{2\pi}{\left(\frac{\pi}{20}\right)} = 2\pi \times \frac{20}{\pi} = 40$ seconds - $\frac{1}{2}$

A 10 minute ride $= 10 \times 60 = 600$ seconds

$\therefore$ Number of rotations $= \frac{600}{40} = 15$ rotations - $\frac{1}{2}$

c) $20\sin\left(\frac{\pi}{20}(t-15)\right) + 22 = 30$

$20\sin\left(\frac{\pi}{20}(t-15)\right) = 8$

$\sin\left(\frac{\pi}{20}(t-15)\right) = \frac{8}{20}$ — 1 mark

$\frac{\pi}{20}(t-15) = 0.4115$ (radians) ↑ 1st quadrant

Some students used degrees.

$t = 0.4115 \times \frac{20}{\pi} + 15$

$t = 17.62$ seconds — 1 mark

Question 34 MARKER'S COMMENTS

d) Method 1

The 2nd time at 30m will be found from the 2nd quadrant solution.

$$\frac{\pi}{20}(t-15) = \pi - 0.4115$$

$$t = \frac{20}{\pi}(\pi - 0.4115) + 15$$

$t = 32.38$ seconds – 1 mark

Given that the passenger stays above 30m between the times of 17.62s and 32.38s, we subtract them to find the time above 30m.

Time above 30m in 1st rotation $= 32.38 - 17.62$

$= 14.76$s

Time above 30m in 15 rotations (10 minutes) $= 14.76 \times 15$

$= 221.4$s

$= 3$ min 41 sec – 1 mark

Students received zero if they made the question too easy from 1st mark.

MARKER'S COMMENTS

Question 34

d) Method 2

- Note: over 30m at $t = 17.62$ s
- Find the time when at the top of the ferris wheel by letting $h = 42$

$$20\sin\left(\frac{\pi}{20}(t-15)\right)+22 = 42$$

$$20\sin\left(\frac{\pi}{20}(t-15)\right) = 20$$

$$\sin\left(\frac{\pi}{20}(t-15)\right) = 1$$

$$\frac{\pi}{20}(t-15) = \frac{\pi}{2}$$

$$t-15 = \frac{\pi}{2}\times\frac{20}{\pi}$$

$$t-15 = 10$$

$t = 25$ seconds. - 1 mark

Time from 30m on way up to 42m $= 25 - 17.62$

$= 7.38$ s

Time above 30m in one rotation $= 7.38 \times 2 = 14.76$ s

Time above 30m in 15 rotations $= 14.76 \times 15$

(10 minutes) $= 221.4$ seconds

$= 3$ min 41 sec - 1 mark

Question 35

MARKER'S COMMENTS

$M = M_0 e^{-kt}$

SUB $M = 30$ when $t = 0$

$30 = M_0 e^{-k(0)}$

$30 = M_0 \times 1$

$\therefore M_0 = 30$ — 1 mark

$\therefore M = 30e^{-kt}$

SUB $M = 15$ when $t = 24$

$15 = 30e^{-k(24)}$

$\frac{1}{2} = e^{-24k}$

$-24k = \ln\left(\frac{1}{2}\right)$

$k = \frac{\ln\left(\frac{1}{2}\right)}{-24}$ — 1 mark

$\therefore M = 30e^{+\left(\frac{\ln\frac{1}{2}}{24}\right)t}$

Find t when $M = 0.1 \times 30 = 3$ kg

$3 = 30e^{+\left(\frac{\ln\frac{1}{2}}{24}\right)t}$ — 1 mark

$e^{\left(\frac{\ln\frac{1}{2}}{24}\right)t} = 0.1$

$\frac{\ln\frac{1}{2}}{24} t = \ln(0.1)$

$t = \frac{\ln(0.1)}{\left(\frac{\ln\frac{1}{2}}{24}\right)} = 79.726$ hours

$\frac{1}{2}$ $\frac{1}{2}$